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SPECTRUM OF INITIAL PERTURBATIONS IN OPEN AND CLOSED INFLATIONARY MODELS

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Abstract. Spectrum of initial scalar and tensor perturbations created during an inflationary stage producing a closed or open FRW universe now is discussed. In the closed case, the CMB temperature anisotropy $\Delta T/T$ generated by scalar perturbations is enhanced for low multipoles. It is argued that in the open case there is no suppression of low multipoles. A possibility of the existence of a preferred space direction in the open case is noted.

1. Introduction.

Though the prediction that total present energy density of matter in the Universe (including the cosmological constant if it is non-zero) should be equal to the critical one $\varepsilon_c = 3H_0^2/8\pi G$ (H_0 is the Hubble constant, and we assume $c = \hbar = 1$) is usually considered as one of the basic predictions of the inflationary scenario of the early Universe, it is not an absolute prediction. More complicated inflationary models can be constructed which contain at least two parameters in effective Lagrangians describing the de Sitter (inflationary) stage and which may lead to $\Omega_m \neq 1$ at present (thus, they belong to the second and higher complexity levels according to the classification of cosmological models presented in [1]). Then the first of the parameters determines an amplitude of the approximately flat ($n_s \approx 1$) spectrum of initial adiabatic perturbations while the second one gives the present value of $\Omega_m = 1 + K/H_0^2 a_0^2$. Here a_0 is the present value of the scale factor of the Friedman-Robertson-Walker (FRW) cosmological model and $K = 1, 0, -1$ denotes closed, flat and open FRW models respectively.

This freedom is due to the fact that the exact de Sitter space-time which serves as a basic element of the inflationary scenario can be covered by different charts (systems of reference). In particular, its metric can be represented as a partial case of all three FRW models:

$$ds^2 = dt_+^2 - H_1^{-2}\cosh^2 H_1 t_+ (d\chi_+^2 + \sin^2\chi_+ d\Omega^2), \quad K = +1, \tag{1}$$

$$ds^2 = dt^2 - a_1^2 e^{2H_1 t}(dr^2 + r^2 d\Omega^2), \qquad a_1 = const, \qquad K = 0, \tag{2}$$

$$ds^2 = dt_-^2 - H_1^{-2}\sinh^2 H_1 t_- (d\chi_-^2 + \sinh^2\chi_- d\Omega^2), \quad K = -1, \tag{3}$$

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2, \quad H_1^2 = \frac{\Lambda_1}{3} = const.$$

Note that the first metric ($K = +1$) covers the whole de Sitter space-time, and the two others have horisons at $t = -\infty$ and $t_- = 0$ respectively. Coordinate transformations between these 3 systems of refernce are given by the following formulas:

a) from (2) to (1)

$$
\begin{aligned}
t &= H_1^{-1} \ln(\cosh H_1 t_+ \cos \chi_+ + \sinh H_1 t_+), \\
r &= (a_1 H_1)^{-1} \frac{\cosh H_1 t_+ \sin \chi_+}{\cosh H_1 t_+ \cos \chi_+ + \sinh H_1 t_+};
\end{aligned}
\tag{4}
$$

b) from (3) to (1)

$$
\begin{aligned}
\cosh H_1 t_- &= \cosh H_1 t_c \cosh H_1 t_+ \cos \chi_+ - \sinh H_1 t_c \sinh H_1 t_+, \\
\sinh H_1 t_- \sinh \chi_- &= \cosh H_1 t_+ \sin \chi_+
\end{aligned}
\tag{5}
$$

where t_c is an arbitrary constant. The metric (3) covers the future light cone of the point $t_+ = t_c$, $\chi_+ = 0$ in the metric (1). Thus, it represents an analog of the Milne metric in the flat space-time. In the inflationary scenario, the de Sitter space-time is neigher exact, nor stable; it is only approximate and metastable. So, by adding homogeneous perturbations to the metrics (1-3) which drive exact solutions away from the de Sitter stage one can obtain a FRW Universe with $\Omega_m \neq 1$ even in the simplest models (see, e.g., Eq.(10) of the paper [2] where the first explicit cosmological model with the initial de Sitter stage was constructed).

However, in that case it requires special fine tuning of initial conditions (namely, of the amplitude of these homogeneous perturbations) at the beginning of inflation to have Ω_m significantly different from 1 at present. In particular, the number of e-folds during inflation should be fine-tuned to about 70. This contradicts the spirit of the inflationary scenario. For natural initial conditions without fine-tuning, the simplest inflationary models predict $\Omega_m \approx 1$ with a high degree of accuracy. Actually, in this case the deviation of Ω_m inside the present cosmological horison from unity is determined by inhomogeneous fluctuations of the quasi-Newtonian gravitational potential inside the horison, too, and not by an isotropic homogeneous part of spatial curvature which is exponentially small. It can be shown that $\Omega_m - 1$ is a Gaussian stochastic quantity with the dispersion

$$
< (\Omega_m - 1)^2 >= \frac{1}{8}(\xi_\Phi(0) - \xi_\Phi(2R_h))
\tag{6}
$$

where ξ_Φ is the potential-potential correlation function and R_h is the horison scale. Numerically, $|\Omega_m - 1|_{rms} \approx 4(\Delta T/T)_Q \approx 3 \cdot 10^{-5}$ where $(\Delta T/T)_Q \approx 7 \cdot 10^{-6}$ is the expected value of the quadrupole anisotropy of the cosmic microwave background (CMB) for $n_s = 1$.

To avoid undesirable fine-tuning of initial conditions, one has eigher to introduce an additional parameter into the inflaton potential or to add a second inflaton scalar field (that leads to double inflation) in order to have 70 e-folds of inflation (or, of the last phase of inflation in the case of double inflation) for typical initial conditions. According to the philosophy used in [1], then the word "fine-tuning" is no more adequate, but one should refer the resulting inflationary model to the second level of complexity.

2. Closed FRW universes

This case is closely related to the hypothesis of the Universe's "creation from nothing" [3-6]. Though this hypothesis still remains unproved (in particular, in no model was

the probability of this process calculated or at least rigorously shown to be non-zero), it is assumed usually that this creation should be described by an instanton (Euclidean) solution of classical equations. An $O(3)$ instanton ($O(4)$, if the energy-momentum tensor of the inflaton field may be approximated by a cosmological constant) just leads to a closed FRW universe in the Lorentzian, or Minkowskian, region. The background equations in the Lorentzian region have the form:

$$H^2 + \frac{1}{a^2} = \frac{8\pi G}{3}\left(\frac{\dot\phi^2}{2} + V(\phi)\right), \quad H \equiv \frac{\dot a}{a},$$

$$\ddot\phi + 3H\dot\phi + \frac{dV}{d\phi} = 0, \tag{7}$$

dot means differentiation with respect to t_+. Equations for instanton configurations in the Euclidean region follow from here after the substitution $t_+ = i\tau$.

Let us assume now that the tunneling ends at the point $\phi = \phi_1$ (so that $\dot\phi = \dot a = 0,\ \phi = \phi_1$ in both the Euclidean and Lorentzian regions at the moment $t_+ = \tau = 0$) lying in a flat region of the inflaton potential (i.e., satisfying the slow-roll conditions $V_1' \equiv \frac{dV}{d\phi}(\phi_1) \ll \sqrt{48\pi G}V_1,\ V_1'' \equiv \frac{d^2V}{d\phi^2}(\phi_1) \ll 24\pi G V_1$ where $V_1 \equiv V(\phi_1)$). Then, just after tunneling an inflationary stage begins. By choosing the number of e-folds between $\phi = \phi_1$ and the end of inflation

$$\ln\frac{a_f}{a_1} = 8\pi G\int_{\phi_f}^{\phi_1} d\phi\, \frac{V(\phi)}{V'(\phi)} \approx 70 \tag{8}$$

(a_f and ϕ_f are the values of the scale factor and the scalar field respectively at the end of inflation), we succeed in constructing of an inflationary model which has $\Omega_m > 1$ at present. Thus, the value ϕ_1 is just the abovementioned second parameter of the inflationary model (the first one defining the amplitude of adiabatic perturbations is H_1^3/V_1' as we shall see below). Of course, the potential $V(\phi)$ should have some specific properties around the point ϕ_1 to facilitate tunneling just to this point, but we shall not discuss this point further.

At the beginning of the inflationary stage $\phi \approx \phi_1$ and $a(t_+) = H_1^{-1}\cosh H_1 t_+,\ H_1^2 = 8\pi G V_1/3$. Then, integrating the second of Eqs. (7), we find

$$\dot\phi = -\frac{V_1'}{a^3}\int_0^{t_+} a^3\, dt = -\frac{V_1'}{3H_1}\tanh^3 H_1 t_+ \left(1 + \frac{3}{\sinh^2 H_1 t_+}\right). \tag{9}$$

For $H_1 t_+ \gg 1$, $\dot\phi$ approaches the standard slow-rolling value $(-V_1'/3H_1)$. So, the subsequent evolution of the background is as in the $K = 0$ case until recent times.

Quantization of a scalar field ψ in the de Sitter background was first performed in [7], and I have nothing to add here. For a massless field ($\psi_{;i}^{;i} = 0$) in the Heisenberg representation,

$$\hat\psi = \sum_{n,l,m}\left(\hat a_{nlm}\psi_n(\eta)Q_{nlm}(\chi_+,\theta,\varphi) + \hat a_{nlm}^+\psi_n^*(\eta)Q_{nlm}^*(\chi_+,\theta,\varphi)\right);$$

$$\left[\hat a_{nlm}\ \hat a_{n'l'm'}^+\right] = \delta_{nn'}\delta_{ll'}\delta_{mm'}, \quad \hat a_{nlm}|0\rangle = 0, \quad |m| \le l < n, \quad n = 1, 2...; \tag{10}$$

$$\triangle Q_{nlm} + (n^2 - 1)Q_{nlm} = 0;$$

$$Q_{nlm} = \sqrt{M_{nl}}\,\frac{P^{-l-1/2}_{n-1/2}(\cos\chi_+)}{\sqrt{\sin\chi_+}}\,Y_{lm}(\theta,\varphi), \quad M_{nl} = \prod_{p=0}^{l}(n^2-p^2); \tag{11}$$

$$\cos\eta = \frac{1}{\cosh H_1 t_+}, \quad a(\eta) = \frac{1}{H_1\cos\eta}, \quad -\frac{\pi}{2} < \eta < \frac{\pi}{2};$$

$$\psi_n(\eta) = \sqrt{\frac{n}{2(n^2-1)}}H_1 e^{-in\eta}\left(\cos\eta + \frac{i\sin\eta}{n}\right). \tag{12}$$

Here P_μ^ν are the Legendre functions and $\triangle$ is the covariant Laplace operator. At the late stage of inflation ($\eta \to \frac{\pi}{2}$), $|\psi_n| \to H_1/\sqrt{2n(n^2-1)}$ as compared to $H_1/(2k^3)^{1/2}$ in the $K=0$ case. So, fluctuations are slightly enhanced at low n if we identify k with n.

Gravitational waves (GW) have the same time dependence [8] with the only difference that $n = 3, 4...$ for them. If the polarization tenzor is normalized by the condition $e_{\alpha\beta}e^{\alpha\beta} = 1$, then the time dependent part $h_n = \sqrt{32\pi G}\psi_n$ [9], so $|h_n|^2 \to \frac{16\pi G H_1^2}{n(n^2-1)}$ for $\eta \to \frac{\pi}{2}$ ($|h_n|^2$ should be understood as the dispersion of a stochastic Gaussian quantity). This gives the initial condition for GW at subsequent FRW power-law stages.

To obtain the spectrum of adiabatic perturbations, one has to use either the equation for the gravitational potential $\Phi = \Psi$ [10]:

$$\ddot\Phi_n + \left(H - 2\frac{\ddot\phi}{\dot\phi}\right)\dot\Phi_n + \left(\frac{n^2-5}{a^2} + 2\dot H - 2\frac{\ddot\phi}{\dot\phi}H\right)\Phi_n = 0, \quad n = 3, 4..., \tag{13}$$

or a master equation for the generalized scalar field perturbation ζ [11]. Φ_n is related to the Lifshitz variables μ_n and λ_n by the formula

$$\Phi_n = -\frac{1}{6}(\mu_n + \lambda_n) + \frac{a\dot a\dot\lambda_n}{2(n^2-1)}. \tag{14}$$

Using any of these equations it is possible to show that Φ_n approaches the standard form at $\eta \to \frac{\pi}{2}$ (i.e., in the region where both the wavelength and the radius of spatial curvature much exceed the Hubble radius):

$$\Phi_n = C_n\left(1 - \frac{H}{a}\int_0^{t_+} a\,dt\right), \quad |C_n| = \frac{3H_1^3}{V_1'}\frac{1}{\sqrt{2n(n^2-1)}}. \tag{15}$$

Therefore, deviation from the flat case for adiabatic perturbations is the same as for gravitational waves: $|C_n| \propto |h_n|$.

When calculating the CMB temperature anisotropy produced by adiabatic perturbations and gravitational waves with these initial spectra in a closed FRW universe, it appears that the main effect comes not so much from the change in the initial spectrum but from an integral term in the Sachs-Wolfe effect resulting from a deviation of a closed matter-dominated FRW universe from a power-law expansion at recent times. Similar to the case of a flat universe with the cosmological constant, it results in noticeable increase of low multipoles in case of adiabatic perturbations [12]. On the other hand, in case of $\Delta T/T$ produced by GW, low multipoles ($l = 2-4$) decrease as compared to the $K=0$ case with the same initial amplitude while higher multipoles slightly increase. However, the magnitude of the effect is significant for the quadrupole only (Q goes down by 15% if $\Omega_m = 2$) [12,13].

Since $V_1' \ll H_1^2/\sqrt{G}$, the relative ratio T/S of tensor and scalar contributions to the large-angle CMB anisotropy is small in a closed inflationary universe. Therefore, the observational prediction is the enhancement of low $\Delta T/T$ multipoles relative to the dependence $C_l \equiv \langle |a_{lm}|^2 \rangle \propto (l(l+1))^{-1}$ which takes place for the flat ($n_s = 1$) spectrum in the $K = 0$ case. In particular, the *rms* value of the quadrupole Q becomes 30% larger for $\Omega_m = 1.5$ and 40% larger for $\Omega_m = 2.0$ if the spectrum is normalized at the $l = 10$ multipole [12]. No such an enhancement is seen in the 2-year COBE data. One may conclude that certainly $\Omega_m < 2$, and probably even $\Omega_m = 1.5$ can be excluded. Thus, no much place for a closed universe remains.

Note that a similar constraint can be obtained without any assumptions about an initial perturbation spectrum, simply from the age of the Universe T. In a closed matter-dominated universe,

$$T = \frac{2}{3H_0}\mathcal{K}_T(\Omega), \quad \mathcal{K}_T(\Omega) = \frac{3}{2(\Omega_m - 1)}\left(\frac{\Omega_m}{2\sqrt{\Omega_m - 1}} \arcsin\frac{2\sqrt{\Omega_m - 1}}{\Omega_m} - 1\right) \tag{16}$$

($\mathcal{K}_T = 1$ for $\Omega_m = 1$). If $H_0 \geq 50$ km/s/Mpc and $T \geq 11$ Gyrs (that seems to be the lowest value permitted by cosmic nucleosynthesis and ages of globular clusters), then $\Omega_m \leq 2.0$. If we raise the lower bound for T to 12 Gyrs, then $\Omega_m \leq 1.45$.

3. Open FRW universes.

As was discussed above, the open chart (3) covers the interior of the future light cone of a 4-point (an event) in the de Sitter space-time. So, one may think of an open inflationary universe as resulting from creation of a bubble of a new de Sitter phase in the old de Sitter phase as a result of the first order phase transition [14,15]. As in the case $K = +1$, the duration of the second de Sitter (inflationary) phase should be fine-tuned to about 70 e-folds. This can be achieved by introducing at least one additional parameter to the effective Lagrangian describing an inflationary stage with the phase transition during it.

We shall further consider the nearly degenerate case with practically equal vacuum energy densities in both phases and neglect the energy density of the bubble wall. In this approximation, the words about the "phase transition" serve only to justify the choice of the perturbation breaking the full de Sitter invariance which depends on the time t_- only. In other words, the de Sitter symmetry breaks to the $O(2,1)$ symmetry. So, we return to the spatially homogeneous decay of the inflationary stage investigated in [2].

Then the problem about inhomogeneous fluctuations of quantum fields generated during this inflationary stage reduces to the quantization of a massless scalar field ψ in the chart (3). One possible consistent quantization may be formally obtained from that in the $K = +1$ case by the change $\chi_+ \to i\chi_-$, $n \to ik$ (k - real), $\eta \to \frac{\pi}{2} - i\eta$. As a result, we get:

$$\hat{\psi} = \sum_{l,m}\int_0^\infty dk(\hat{a}_{klm}\psi_k(\eta)Q_{klm}(\chi_-,\theta,\varphi) + \hat{a}_{klm}^+\psi_k^*(\eta)Q_{klm}^*(\chi_-,\theta,\varphi));$$

$$\left[\hat{a}_{klm}\ \hat{a}_{klm}^+\right] = \delta(k-k')\delta_{ll'}\delta_{mm'}, \quad a_{klm}|\tilde{0}\rangle = 0; \tag{17}$$

$$\triangle Q_{klm} + (k^2+1)Q_{klm} = 0;$$

$$Q_{klm} = \sqrt{N_l}\ \frac{P_{ik-1/2}^{-l-1/2}(\cosh\chi_-)}{\sqrt{\sinh\chi_-}}\ Y_{lm}(\theta,\varphi), \quad N_l = \prod_{p=0}^{l}(k^2+p^2); \tag{18}$$

$$\sinh|\eta| = \frac{1}{\sinh H_1 t_-}, \quad a(\eta) = \frac{1}{H_1 \sinh|\eta|}, \quad -\infty < \eta < 0;$$
$$\psi_k(\eta) = \sqrt{\frac{k}{2(k^2+1)}} H_1 e^{-ik\eta} \left(\sinh\eta - \frac{i\cosh\eta}{k} \right). \tag{19}$$

This leads to *rms* values of fluctuations produced at the end of the de Sitter stage ($\eta \to 0$): $(\psi_k)_{rms} = H_1/\sqrt{2k(k^2+1)}$ [16]. However, due to the fact that $t_- = 0$ ($\eta = -\infty$) is the (particle) horizon and from the analogy with the quantization in the flat space-time in the Milne metric, we know that $|\tilde{0}\rangle$ is not the correct "vacuum" state! Moreover, it can be shown that the average energy-momentum tensor of the field ψ is not regular at $\eta \to -\infty$.

An additional complication follows from the fact that the hypersurface $t_- = const$ is not the Cauchy hypersurface of the full de Sitter space-time. Thus, we cannot assume that perturbations are regular or square integrable at $\chi_- \to \infty$. As a result, one should either abandon the assumption that different modes are uncorrelated (which follows from the commutator condition (17)), or add some terms not included into the complete orthonormal set (18) which are determined by a boundary condition at the horizon. The de Sitter-invariant quantization of a massive scalar field performed in [17] shows that in the massless limit one has to add the $k^2 = -1$ mode. However, some divergences remain in the case of tensor perturbations (gravitational waves) [18], so the question is far from being clear.

On the other hand, even the assumption of the de Sitter invariance of the Heisenberg quantum state is not justified in this case because the chart (3) covers only a part of the de Sitter space-time. So, here we shall use a different approach to calculate fluctuations of a test massless scalar field generated during the inflationary stage in the open chart (3). This test field may serve, e.g., as a toy model for isocurvature perturbations.

The idea is simply to take the Green function of the massless scalar field ψ for a real inflationary stage that begins at the moment $t = 0$ in the flat chart (2) (or at the moment $t_+ = 0$ in the full chart (1) - this makes no difference as we shall see) and to reduce it to the chart (3) using the formulas (4,5). It is known [19-21] that this Green function is not de Sitter invariant:

$$G(t_A, \vec{r}_A; t_B, \vec{r}_B) \equiv <\psi(t_A, \vec{r}_A)\psi(t_B, \vec{r}_B)> = \frac{H_1^3}{8\pi^2}(t_A + t_B) - \frac{H_1^2}{8\pi^2}\ln|z_{AB}| + const; \tag{20}$$

$$z_{AB} = \cosh s_{AB} = \cosh(H_1(t_A - t_B)) - \frac{a_1^2}{2} e^{H_1(t_A+t_B)} |\vec{r}_A - \vec{r}_B|^2, \qquad K = 0,$$
$$z_{AB} = \cosh H_1 t_{-A} \cosh H_1 t_{-B} - \sinh H_1 t_{-A} \sinh H_1 t_{-B} \cosh\zeta_{AB}, \qquad K = -1, \tag{21}$$
$$\cosh\zeta_{AB} = \cosh\chi_{-A}\cosh\chi_{-B} - \sinh\chi_{-A}\sinh\chi_{-B}\cos\theta_{AB} \tag{22}$$

where s_{AB} is the geodesic distance between the 4-points A and B , ζ_{AB} is the $3D$ geodesic distance between the 3-points $(\chi_{-A}, \theta_A, \varphi_A)$ and $(\chi_{-B}, \theta_B, \varphi_B)$ and θ_{AB} is the angle between unit 3-vectors with the angular directions (θ_A, φ_A) and (θ_B, φ_B).

We assume that the first phase of inflation before the bubble formation is long: $H_1 t_c \gg 1$. Then in the limit $H_1 t_- \gg 1$ corresponding to the end of the second inflationary phase, the coordinate transformation from the $K = 0$ case to $K = -1$ case reduces to:

$$r = r_c \tanh\frac{\chi_-}{2}, \quad r_c = 2(a_1 H_1 e^{H_1 t_c})^{-1},$$
$$H_1(t - t_c) = H_1 t_- + \ln\frac{1 + \cosh\chi_-}{2}. \tag{23}$$

Substituting (23) into (20), we obtain the answer:

$$G(t_{-A}, \vec{r}_A; t_{-B}, \vec{r}_B) = \frac{H^2}{8\pi^2} \ln \frac{(1+\cosh\zeta_{AC})(1+\cosh\zeta_{BC})}{\cosh\zeta_{AB} - 1} + const, \tag{24}$$

where C is the point $\chi_- = 0$ - the "center" of the open universe. The Green function (24) is stationary, this means that fluctuations are time-independent outside the horizon as they should be. On the other hand (and this is a new and unexpected result), it is neither translationally invariant, nor isotropic. So, we get a spontaneous breakdown of homogeneity and isotropy even in the statistical sence.

Since we didn't specify the nature of the field ψ, it may appear that ψ itself is not observable, and only its differences can be measured (e.g., this takes place in case of the gravitational potential Φ). Then let us introduce the observable quantity $\tilde{\psi} = \psi(\vec{r}) - \psi(\vec{r}_O)$ where $\vec{r}_O$ is the observer (i.e., our) location. The correlation function of $\tilde{\psi}$ has the form:

$$\tilde{G} \equiv < \tilde{\psi}(\vec{r}_A)\tilde{\psi}(\vec{r}_B) > = \frac{H^2}{8\pi^2} \ln \frac{(\cosh\zeta_{AO} - 1)(\cosh\zeta_{BO} - 1)}{\cosh\zeta_{AB} - 1} + const. \tag{25}$$

It is isotropic with respect to the observer, but observer-dependent clearly. Suppose that ψ directly produces $\Delta T/T$ at the last scattering surface. Then, shifting the center of coordinates to the observer location and taking $\chi_{-A} = \chi_{-B} = \chi_{hor}$, we get the correlation function $G(\theta)$:

$$G(\theta) = \frac{H^2}{8\pi^2}\left(\ln \frac{1}{1-\cos\theta_{AB}} + const\right). \tag{26}$$

This $G(\theta)$ just coinsides with that produced by adiabatic perturbations with the flat ($n_s = 1$) spectrum in the $K = 0$ case, the corresponding dispersion of multipoles is $C_l \propto (l(l+1))^{-1}$. Thus, in open inflationary models we expect no damping of low multipoles with $l < \Omega_m^{-1}$, contrary to what happens in genuine open FRW models [22]. Actually, when the integral term in the Sachs-Wolfe effect is taken into account, low multipoles will be additionally amplified. Therefore, the total expected effect for $\Delta T/T$ multipoles in an open inflationary universe is qualitatively the same as in a closed inflationary universe: enhancement of low multipoles above the law $C_l \propto (l(l+1))^{-1}$. From the absence of such an enhancement in the COBE data, restrictions on Ω_m can be obtained which we will not discuss here. Note also that the anisotropy of the correlation function G (24) may become observable in models where H is not constant during the first phase of inflation. However, this effect will be proportional to $(n_s - 1)$ and rather small.

So, the final conclusion is that there are still some place for inflationary models with $\Omega_m \neq 1$ though we don't see any specific observational effect (e.g., enhancement of low multipoles in $\Delta T/T$ above the level predicted by standard inflation) which would push us to accept these models.

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REFERENCES

1. A.A. Starobinsky, this volume.
2. A.A. Starobinsky. Phys. Lett., **B91**, 99, 1980.
3. E.P. Tryon. Nature, **246**, 396, 1973.

4. P.I. Fomin. Preprint ITF-73-137, Kiev, 1973; Dokl. Akad. Nauk. Ukr. SSR, **A9**, 831, 1975.
5. Ya.B. Zeldovich. Sov. Astron. Lett., **7**, 322, 1981.
6. L.P. Grishchuk, Ya.B. Zeldovich. In: *Quantum structure of space-time*, eds. M. Duff and C.I. Isham, Cambridge Univ. Press, p 409, 1982.
7. N.A. Chernikov, E.A. Tagirov. Ann. Inst. Henri Poincaré, **9A**, 109, 1968.
8. E.M. Lifshitz. Zh. Eksp. Teor. Fiz., **16**, 587, 1946.
9. A.A. Starobinsky. JETP Lett., **30**, 682, 1979.
10. J.J.Halliwell, S.W. Hawking. Phys. Rev. D, **31**, 1777, 1985.
11. K.K. Olkhin, this volume.
12. T. Souradeep, A.A. Starobinsky, in preparation.
13. B. Allen, R. Caldwell, S. Koranda. Preprint astro-ph/9410024, 1994.
14. J.R. Gott, III. Nature, **295**, 304, 1982.
15. M. Bucher, A.S. Goldhaber, N. Turok. Preprint hep-ph/9411206, 1994.
16. D.Lyth, E.D. Stewart. Phys. Lett., **B252**, 336, 1990.
17. M. Sasaki, T. Tanaka, K. Yamamoto. Phys. Rev. D, **51**, 2979, 1995.
18. B. Allen, private communication.
19. A.D. Linde. Phys. Lett., **B116**, 335, 1982.
20. A.A. Starobinsky. Phys. Lett., **B117**, 175, 1982.
21. A. Vilenkin, L.H. Ford. Phys. Rev. D, **26**, 1231, 1982.
22. M.L. Wilson. Astroph. J., **273**, 2, 1983.